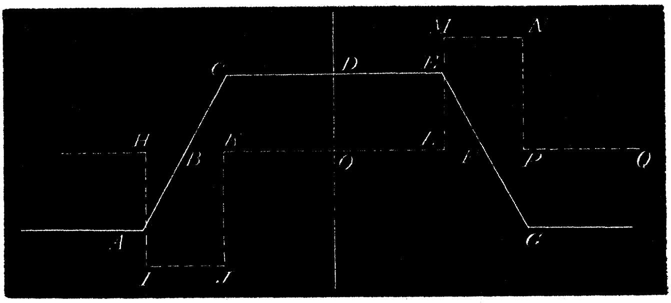
FIG. 3.



resulting relations of potential in the secondary and the time will be indicated by the dotted line HIJKOLMNPQ. The mean square observed will be proportional to ML.$\sqrt{}$LP; but ML.LP is proportional to EL, hence the potential observed will vary inversely as $\sqrt{}$LP, even though the maximum induction remain constant. If then the maximum induction be deduced on the assumption that the induction is a simple harmonic function of the time, results may readily be obtained vastly in excess of the truth.

II. "Note on the Theory of the Alternate Current Dynamo." By JOHN HOPKINSON, M.A., D.Sc., F.R.S. Received February 17, 1887.

According to the accepted theory of the alternate current dynamo, the equation of electric current in the armature is $\gamma\dot{y}+Ry=$ periodic function of t, where γ is a constant coefficient of self-induction. This equation is not strictly true, inasmuch as γ is not in general constant,* but it is a most useful approximation. My present purpose is to indicate how the values of γ and of the periodic function representing the electromotive force can be calculated in a machine of given configuration.

To fix ideas, we will suppose the machine considered to have its magnet cores arranged parallel to the axis of rotation, that the cores are of uniform section, also that the armature bobbins have iron cores, so that we regard all the lines of induction as passing either through an armature coil, or else between adjacent poles entirely outside the armature. The sketch shows a development of the machine considered. The iron is supposed to be so arranged that the currents

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* "On the Theory of Alternating Currents," 'Telegr. Engin. Journ.,' vol. 13, 1884, p. 496.

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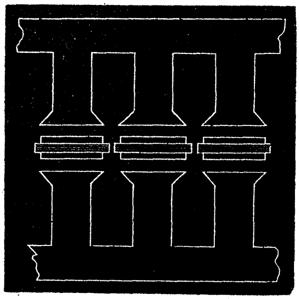

induced therein may be neglected. We further suppose for simplicity that the line integral of magnetic force within the armature core may be neglected.

Let A_1 be the effective area of the space between the pole piece and armature core when the cores are in line, l_1 the distance from iron to iron.

Let A_2 be the section of magnet core, l_2 the effective length of a pair of magnet limbs, so that l_2 may be regarded as the length of the lines of force as measured from one pole face to the next.

Let m be the number of convolutions in a pair of magnet limbs, and

n, the convolutions in one armature section,

T, the periodic time.

The time is measured from an epoch when the armature coil we shall consider is in a symmetrical position in a field which we shall regard as positive.

x and y are the currents in the magnet and armature coils, the positive direction being that which produces the positive field at time zero.

At time t the armature coil considered has area A_1',

$$= b_0 + b_1 \cos(2\pi t/\mathrm{T}) + b_2 \cos(4\pi t/\mathrm{T}) + \&c.$$

in a positive field; and area A_1'',

$$= b_0 - b_1 \cos(2\pi t/\mathrm{T}) + b_2 \cos(4\pi t/\mathrm{T}) - \&c.$$

in a negative field, where

$$b_0 + b_1 + b_2 + \ldots = A_1,$$

and

$$b_0 - b_1 + b_2 + \ldots = 0.$$

The coefficients b_0, b_1, &c., are deducible by Fourier's theorem from a drawing of the machine under consideration.

Let I be the total induction in the magnet core, and let at time t I be distributed into I$'$ through A_1', I$''$ through A_1'' and I$'''$ as a waste field to the neighbouring poles.

The line integral of magnetic force from the pole to either adjacent pole is I'''/k, where k is a constant.

We have first to determine I', I'', I''', in terms of x and y.

Take the line integral of magnetic force in three ways through the magnets, and respectively through area A_1', through area A_1'', and across between the adjacent poles—

$$l_2 f\left(\frac{I}{A_2}\right)+2l_1\frac{I'}{A_1'} = 4\pi m x+4\pi n y,$$

$$l_2 f\left(\frac{I}{A_2}\right)+2l_1\frac{I''}{A_1''} = 4\pi m x-4\pi n y,$$

$$l_2 f\left(\frac{I}{A_2}\right)+\frac{I'''}{k} = 4\pi m x;$$

whence $$I'+I''+I''' = I$$

$$= \left(\frac{A_1'+A_1''}{2l_1}+k\right)\left(4\pi m x-l_2 f\left(\frac{I}{A_2}\right)\right)+\frac{A_1'-A''}{2l_1}\cdot 4\pi n y.$$

When t, x, and y are given, this would suffice to determine I by means of the known properties of the material of the magnets as represented by the function f. We will, however, consider two extreme cases between which other cases will lie.

First.—Suppose that the intensity of induction in the magnet cores is small, so that $l_2 f(I/A_2)$ may be neglected, the iron being very far from saturation. We have—

$$I'-I'' = \frac{4\pi}{2l_1}\left\{m(A_1'-A_1'')x+n(A_1'+A_1'')y\right\}$$

$$= \frac{4\pi}{l_1}\left\{m\left(b_1\cos\frac{2\pi t}{T}+b_3\cos\frac{6\pi t}{T}+\dots\right)x\right.$$

$$\left.+n\left(b_0+b_2\cos\frac{4\pi t}{T}+\dots\right)y\right\}.$$

We see that the coefficient of self-induction γ in general contains terms in $\cos(4\pi t/T)$.

Second.—In actual work it would be nearer the truth to suppose that the magnetising current x is so great that the induction I may be regarded as constant, and the quantity $l_2 f(I/A_2)$ as considerable. But as small changes in I imply very great changes in $l_2 f(I/A_2)$, its value cannot be regarded as known. We have then—

$$2l_1\frac{\mathrm{I}'}{\mathrm{A}_1'} = \frac{2l_1}{\mathrm{A}_1'+\mathrm{A}_1''+2kl_1}\left(\mathrm{I}-\frac{\mathrm{A}_1'-\mathrm{A}_1''}{2l_1}\,.\,4\pi my\right)+4\pi my,$$

$$2l_1\frac{\mathrm{I}''}{\mathrm{A}_1''} = \frac{2l_1}{\mathrm{A}_1'+\mathrm{A}_1''+2kl_1}\left(\mathrm{I}-\frac{\mathrm{A}_1'-\mathrm{A}_1''}{2l_1}\,.\,4\pi my\right)-4\pi ny,$$

whence

$$\mathrm{I}'-\mathrm{I}'' = \frac{(\mathrm{A}_1'-\mathrm{A}_1'')\mathrm{I}}{\mathrm{A}_1'+\mathrm{A}_1''+2kl_1}-\left\{\frac{(\mathrm{A}_1'-\mathrm{A}_1'')^2}{\mathrm{A}_1'+\mathrm{A}_1''+2kl_1}-(\mathrm{A}_1'+\mathrm{A}_1'')\right\}\frac{4\pi ny}{2l_1}$$

$$= \frac{(\mathrm{A}_1'-\mathrm{A}_1'')\mathrm{I}}{\mathrm{A}_1'+\mathrm{A}_1''+2kl_1}+\frac{4\mathrm{A}_1'\mathrm{A}_1''+2kl_1(\mathrm{A}_1'+\mathrm{A}_1'')}{\mathrm{A}_1'+\mathrm{A}_1''+2kl_1}\,.\,\frac{4\pi ny}{2l_1}.$$

For illustration, consider the simplest possible case: let $b_0 = b_1 = \frac{1}{2}\mathrm{A}_1$, and $b_2 = b_3 = \ldots = 0$, and let $2kl_1$ be negligible; we have—

$$\mathrm{I}'-\mathrm{I}'' = \mathrm{I}\cos\frac{2\pi t}{\mathrm{T}}+\mathrm{A}_1\sin^2\frac{2\pi t}{\mathrm{T}}\,.\,\frac{4\pi ny}{2l_1},$$

and the equation of current will be—

$$\mathrm{R}y = n\left\{\frac{2\pi}{\mathrm{T}}\mathrm{I}\sin\frac{2\pi t}{\mathrm{T}} - \frac{2\pi n\mathrm{A}_1}{l_1}\frac{d}{dt}\left(\sin^2\frac{2\pi t}{\mathrm{T}}\cdot y\right)\right\},$$

instead of the simple and familiar linear equation.

III. "Transmission of Sunlight through the Earth's Atmosphere." By Captain W. DE W. ABNEY, R.E., F.R.S. Received February 17, 1887.

(Abstract.)

The observations were made by means of the colour photometer which General Festing and himself introduced last year, and which they described in the Bakerian Lecture for 1886. They extended over more than a year, the object being to ascertain the intensity of the different rays in the solar spectrum after passing through various thicknesses of the atmosphere. Owing to the unpromising results obtained by Langley with his bolometer experiments, it was not anticipated that the variation in the intensities of the different rays would obey any law, but subsequent investigation showed that as a rule the intensity of any ray obeyed the law enunciated by Lord Rayleigh, in that $\mathrm{I}' = \mathrm{I}\,\epsilon^{-kx\lambda^{-4}}$, where I and I′ are the initial and transmitted wave-lengths, x the thickness of the medium through which the ray passed, and k a constant, λ being the wave-length. The